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A Model of Mass and Energy Flow in Integrated Biomass Systems

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ABSTRACT

A model based on mass and energy flow is proposed to describe the dynamic behavior of a six-compartment system that integrates the novel uses of biomass for the production of energy and organics with conventional agricultural, agro-industrial and forestry activities. The model is used in the investigation of the system's possible equilibria and their stability, and the corresponding equilibrium and stability conditions are specified and discussed. The effects of biomass utilization on the overall dynamics of the system are analyzed, and the relation of biomass availability to equilibrium and stability is determined. It is concluded that, under certain conditions, the incorporation of novel biomass uses can significantly promote the integration of plant, animal and human populations. The potential of the model is illustrated by an application to the biosystem of Greek agriculture for the period 1960–73.

Key words: Biomass, ecological modeling, mass and energy flow, equilibrium, stability, agro-industrial system, recycling.

NOMENCLATURE

a_{ij}	interaction coefficients between levels i and j; eqn (3a–c)
A_i, A_i'	functions of coefficients a_{ij}, p_i; eqns (17) and (25)
b	eigenvalues
B_{ij}	functions of coefficients a_{ij}, p_i, q_i; eqn (27)
$\dot{e}_{ij}$	flow of energy from level i to level j
E_i	energy at compartment i
$F_i(M)$	functions of a_{ij}, p_i, q_i and M_j $(i \neq j)$, eqn (26)
$\dot{m}_{ij}$	flow of mass from level i to level j

M_i	biomass at compartment i
M_i^*	equilibrium value of M_i
p_i, q_i	coefficients of biological recycle flow from level i; eqn (3d)
t	time
ε_i	process energy losses at level i
ε'_i	process energy requirements at level i
λ_i	energy content of M_i
λ_{ij}	energy content of $\dot{m}_{ij}$
ρ_i	functions of coefficients a_{12}, q_1, q_2; eqn (15)

Subscripts

i, j, l	level, subsystem or compartment (0, 1, ..., 5)
s	system (unspecified destination)
x	external source or destination

Superscript

D	biological recycling

INTRODUCTION

Bioresources possess a unique position among the various renewable sources of energy and chemicals, since their production, availability and end-use are strongly related to the existing food and materials networks. The question of integrating any novel applications of biomass to the conventional ones becomes, therefore, a major element in bioresources assessment studies at the local, regional and national level.[1-3] Systems approach appears particularly promising in the analysis of this type of problem. Integrated biomass systems — the objects to be analyzed — are composed of a large number of 'organic' activities: conventional, e.g. production of food, feed and materials, and novel, e.g. production of biofuels and biochemicals.

A report by Rexen and Munck[4] provides a good example of such an integrated system structured around the question of utilization of cereal crops in Europe. The main components in the system are agriculture and various food and non-food industries, i.e. the animal feed, the cellulose–paper–board, the textile, the polymer, the chemical and the pharmaceutical industries. The systems in question are, in essence, enlarged forms of agro-industrial ecosystems, hence their modeling as well as their scientific management could significantly benefit from the corresponding experience at the interface of theoretical ecology with systems of science and engineering.[5-8]

Of particular value is the modeling approach based on the analysis of mass and energy flows in compartmental ecosystems, as originally developed by Ulanowicz,[9] May[10] and Smerage,[11] and appropriately modified by Hirata and his associates to make the investigation of specific production systems possible.[12-16] In a parallel effort, Hannon, Constanza and others attempted to relate the economics and energetics of production to the structure of the ecosystems involved by using input–output models.[17-22] Both approaches can be fruitful in the analysis of integrated biomass systems, the latter in correlating the technical to the economic aspects,[23] the former in elucidating the basic structure and dynamics of such complex, multi-sectoral networks.[24]

The object of this paper is the formulation of a mathematical model describing the flows of mass and energy in integrated biomass systems. This model will be developed in the case of an integrated biosystem, which consists of six interacting compartments or levels (subsystems). The modeling approach adopted here follows the work of Hirata and Fukao.[12,13] Ecosystems modeling is considered in this paper as a pre-requisite for ecosystem management; therefore, the analysis will focus on the critical questions of equilibrium and stability and the potential effects of biomass utilization on the dynamics of the system.

THE BIOMASS SYSTEM

The biomass network to be modeled in this work is presented in Fig. 1; to simplify this generalized view of the system, we have shown only the mass flows. The mass flow rates from level i to level j are indicated by $\dot{m}_{ij}$.

The system contains six compartments corresponding to the major forms of 'organic' activities in a community as follows:

- Level 0 is the ground level of the system, namely a nutrient and energy pool including biological decomposers, e.g. as in top soil;
- Level 1 is characterized by plant-biomass production through agriculture and/or forestry;
- Level 2 is where animal biomass is produced by raising animals on grazing and market feeds;
- Level 3 includes the conversion of biomass to food, feed and organic materials by conventional agro- and forest industry;
- Level 4 corresponds to the conversion of bioresources to biofuels, biochemicals and biomaterials by novel bio-industries; and
- Level 5 incorporates the relevant activities of the human community with its demand for food, fuel and other organic commodities.

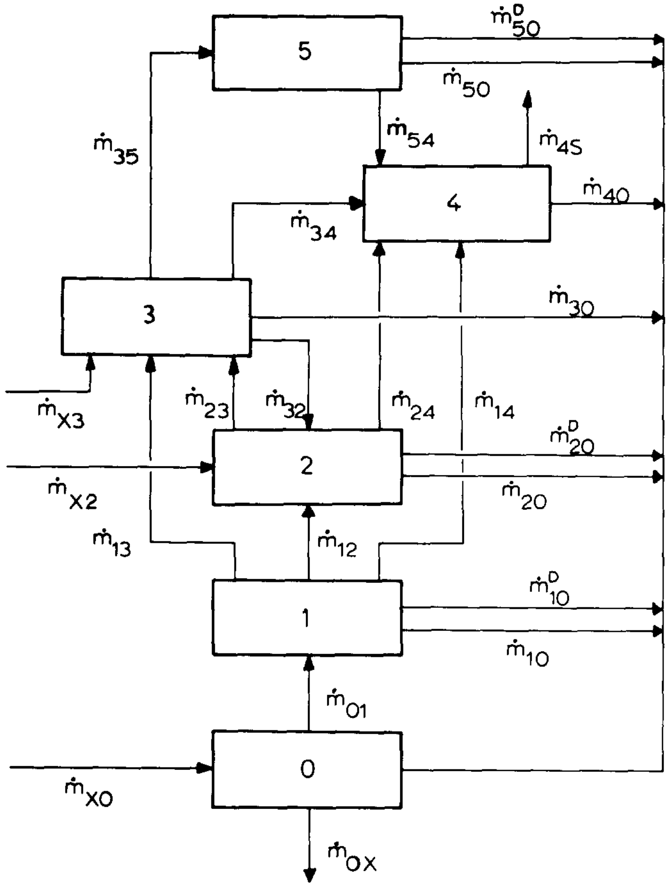

Fig. 1. The integrated biomass system with its associated network of mass flows. The explanation of the various symbols is given in the text.

We note that the only level where primary production of biomass takes place in the system is level 1; at that compartment, inorganic carbon plus the necessary nutrients ($\dot{m}_{01}$) are photosynthetically converted to plant matter. That phytomass serves as an input to levels 2–5, which represent various types of biomass consumers, in a broader use of the term.[25] Secondary biomass and/or waste production takes place at each consumer subsystem as a result of the corresponding biological and technical conversion processes.

The following characteristics of the consumer network, as depicted in Fig. 1, must be noted:

(a) The animal subsystem (level 2) is an aggregated expression of the production processes associated with the systematic raising of domesticated herbivorous animals; secondary biomass production is based on the consumption of three major types of animal feed: grazing ($\dot{m}_{12}$), processed materials ($\dot{m}_{32}$) and imported products ($\dot{m}_{x2}$).

(b) The compartment of the conventional agro- and forest industry (level 3) also plays the role of a reservoir for the flow of biomass via the established channels in the forms of plant, animal and imported products ($\dot{m}_{13}$, $\dot{m}_{23}$ and $\dot{m}_{x3}$, respectively); the received biomass is then used, after processing, to satisfy the needs of the animal husbandry and human community in food and other required organics ($\dot{m}_{32}$ and $\dot{m}_{35}$, respectively).

(c) A basic assumption of the model is that the human demand for biomass-derived commodities (level 5) is satisfied solely through level 3; this corresponds to the existence of a market-regulated economy.

(d) The bio-industry subsystem (level 4) is supplied with bioresources from levels 1, 2, 3 and 5 ($\dot{m}_{14}$, $\dot{m}_{24}$, $\dot{m}_{34}$ and $\dot{m}_{54}$, respectively); various organic wastes and residues but also energy and multipurpose crops are among the major inputs. The products ($\dot{m}_{4s}$, not further analyzed at this stage of the investigation) include biofuels (e.g. alcohols, biogas, refined solid fuels), biochemicals (bulk and fine chemicals, pharmaceuticals), single-cell-protein, organic fertilizer, biopolymers and alternative forms of fiber. Overall, this compartment behaves like a biomass refinery[26] associated with the system.

According to the model, the system communicates with the outside world through the following mass flows:

an inflow of inorganic carbon, e.g. as CO_2, and nutrients, e.g. as imported fertilizer, to level 0 ($\dot{m}_{x0}$);

an inflow of animal feed and other required organics, as imports to level 2 ($\dot{m}_{x2}$);

an inflow of various bioresources, e.g. imports of food, fiber and other organic raw materials, to level 3 ($\dot{m}_{x3}$); and

an outflow (loss) of carbon and nutrients from level 0 ($\dot{m}_{0x}$).

The above defined flows are always positive, i.e. they have the direction indicated in Fig. 1, with the exception of $\dot{m}_{x3}$, which expresses the net bioresources imports and therefore can be either positive or negative, depending on the relative values of imports and exports.

Carbon and nutrients recycling to ground level is a vital component of the flow network. We can distinguish two types of recycling:

intrinsic or biological recycling due to death, loss and litter from the living subsystems ($\dot{m}_{i0}^{D}$, $i=1, 2, 5$); and
extrinsic or technical recycling of residues, wastes and other organic materials from all levels ($\dot{m}_{i0}$, $i=1, \ldots, 5$).

The preceding description of the mass flows is also valid for the energy flows, when the quantities $\dot{m}_{ij}$ are substituted by $\dot{e}_{ij}$, the latter term representing the energy flow from level i to level j. There are two forms of energy flow in this system: energy contained in the flowing mass, and energy inputs required by the various subsystems. Both forms of energy flow are strongly related to the corresponding mass flows.

THE MODEL

According to the modeling approach adopted in this work,[8,9,11,12] the mass flow balance equations take the following form:

$$\frac{\mathrm{d}M_i}{\mathrm{d}t}=\sum_j \dot{m}_{ji}-\sum_i \dot{m}_{ij}+\dot{m}_{xi}-\dot{m}_{i0}^{D} \tag{1a}$$

$$\frac{\mathrm{d}M_0}{\mathrm{d}t}=\sum_i m_{i0}+\sum_i m_{i0}^{D}+\dot{m}_{x0}-\dot{m}_{01}-\dot{m}_{0x} \tag{1b}$$

where $i=1, \ldots, 5$, $j=0, 1, \ldots, 5$, $i \neq j$; M_0, M_i are the biomasses associated with the functioning of each compartment as follows:

M_0 is the total mass of nutrients plus the total biomass of the system's decomposers (e.g. soil microorganisms);
M_1 is the total plant biomass produced over a certain period of time in the system, including both net photosynthetic production and fluctuation of standing biomass;
M_2 is the total animal population in the system, expressed in the appropriate biomass equivalent;
M_3 and M_4 are the total quantities of biomass processed at levels 3 and 4, respectively, over the same period of time as M_1; and
M_5 is the total human population in the system, expressed in its biomass equivalent.

In the usual case when the period of reference for M_1, M_3 and M_4 is one year, then M_0, M_2 and M_5, are estimated as yearly average values.

The energy flow balance equations can be written in a similar way:

$$\frac{\mathrm{d}E_i}{\mathrm{d}t}=\sum_j \dot{e}_{ji}-\sum_i \dot{e}_{ij}+\dot{e}_{xi}-\dot{e}_{i0}^{D} \tag{2a}$$

$$\frac{dE_0}{dt}=\sum_i \dot{e}_{i0}+\sum_i \dot{e}_{i0}^{D}+\dot{e}_{x0}-\dot{e}_{01}+\dot{e}_{0x} \quad (2b)$$

where i and j are as above; E_0, E_i are the energy 'contents' of the six compartments. The various terms of the energy flow equations (2) are linearly related to the corresponding terms of the mass flow equations (1). As shown in Appendix I, those linear relationships make possible the expression of the energy quantities E_i and $\dot{e}_{ij}$ in terms of the mass quantities M_i and $\dot{m}_{ij}$.

Further development of the model therefore depends on the appropriate mathematical expression of the mass flow terms $\dot{m}_{ij}$. As indicated in Appendix I, several types of simple relationships are possible.

$\dot{m}_{ij}=a_{ij}M_i$, when i is dominant (donor control) (3a)

$\dot{m}_{ij}=a_{ij}M_j$, when j is dominant (recipient control) (3b)

$\dot{m}_{ij}=a_{ij}M_iM_j$, when neither level is dominant (mutual control) (3c)

As we can see in eqn (24), the interaction parameters a_{ij} ($i, j = 0, 1, \ldots, 5, x$) are lumped expressions of several ecological and technical factors; the dominance effect in question has to be decided for each flow on economic and ecological grounds. By following this type of reasoning, in the case of the integrated plant–animal–human community system described by Fig. 1, we obtain the following:

Donor control: $\dot{m}_{0x}$, $\dot{m}_{13}$, $\dot{m}_{14}$, $\dot{m}_{10}$, $\dot{m}_{23}$, $\dot{m}_{24}$, $\dot{m}_{20}$, $\dot{m}_{34}$, $\dot{m}_{30}$, $\dot{m}_{4s}$, $\dot{m}_{40}$, $\dot{m}_{54}$ $\dot{m}_{50}$;
Recipient control: $\dot{m}_{01}$, $\dot{m}_{x2}$, $\dot{m}_{x3}$, $\dot{m}_{32}$, $\dot{m}_{35}$, $\dot{m}_{x0}(M_1)$;
Mutual control: $\dot{m}_{12}$.

Based on the assumptions of this work, all a_{ij} have positive values with the exception of a_{x3}.

The biological recycle flows are characterized by a non-linear form of donor control:

$$\dot{m}_{i0}^{D}=p_iM_i+q_iM_i^2, \quad i=1, 2, 5, q_i>0 \quad (3d)$$

The introduction of quadratic density effects[12] was found necessary to guarantee the self-stabilizing intrinsic behavior of the living sectors in the absence of any other interaction. As observed by Lotka,[27] the above is the simplest mathematical expression that can describe the growth of a population with a stable upper limit.

By applying relationships (3a–d) to eqns (1), we arrive at the final form of the model for the particular case considered here:

$$\frac{dM_1}{dt}=A_1M_1-a_{12}M_1M_2-q_1M_1^2 \quad (4a)$$

$$\frac{dM_2}{dt} = A_2M_2 + a_{12}M_1M_2 - q_2M_2^2 \tag{4b}$$

$$\frac{dM_3}{dt} = A_3M_3 - F_3(M) \tag{4c}$$

$$\frac{dM_4}{dt} = -A_4M_4 + F_4(M) \tag{4d}$$

$$\frac{dM_5}{dt} = A_5M_5 - q_5M_5^2 \tag{4e}$$

$$\frac{dM_0}{dt} = A_0M_1 - a_{0x}M_0 + F_0(M) \tag{4f}$$

where the quantities A_i are lumped expressions of the coefficients a_{ij}, p_i, q_i; and $F_j(M)$ are functions of the biomasses M_i $(i \neq j)$, as presented in Appendix II.

Equations (4) are mass-rate expressions; however, they were derived from both mass and energy balance equations. Therefore, the model can be used to investigate the structure and dynamics of the system, and not just of the mass flow network, as long as the linearities of eqn (20) are valid (Appendix I).

EQUILIBRIUM AND STABILITY CONDITIONS

The above developed model will be used to investigate the conditions under which the system can reach an equilibrium or steady state and to determine the equilibrium values of M_i $(i = 0, 1, \ldots, 5)$. Since at equilibrium the left-hand side of eqns (4) is zero, the equations can be solved for the unknown equilibrium values. If M_i^* are the non-zero solutions, then

$$M_1^* = \frac{q_2A_1 - a_{12}A_2}{a_{12}^2 + q_1q_2} \tag{5a}$$

$$M_2^* = \frac{q_1A_1 + a_{12}A_1}{a_{12}^2 + q_1q_2} \tag{5b}$$

$$M_3^* = \frac{F_3^*(M)}{A_3} \tag{5c}$$

$$M_4^* = \frac{F_4^*(M)}{A_4} \tag{5d}$$

$$M_5^* = \frac{A_5}{q_5} \tag{5e}$$

$$M_0^* = \frac{A_0 M_1^* + F_0^*(M)}{a_{0x}} \tag{5f}$$

where $F_j^*(M)$ are the values of the same functions $F_j(M)$ as in eqns (4) for $M_i = M_i^*$.

The solutions shown in eqn (5) have to be positive, hence the coefficients a_{ij}, p_i, q_i must satisfy the following equilibrium constraints:

$$A_1 > 0 \tag{6}$$

$$\frac{q_2}{a_{12}} > \frac{A_2}{A_1} > -\frac{a_{12}}{q_1} \tag{7}$$

$$A_3 F_3^*(M) > 0 \tag{8}$$

$$A_5 > 0 \tag{9}$$

$$A_0 > -\frac{F_0^*(M)}{M_1^*} \tag{10}$$

There is no equilibrium condition specifically for level 4; due to the assumptions of the model, the novel sector of biomass utilization will always reach a non-zero equilibrium, provided that the sectors 1, 2, 3 and 5, which supply the biomass, reach one.

The non-zero state described by relationships (5) is one of the nine possible equilibria, as shown in Table 1. A zero equilibrium value for the biomass in a subsystem means that at equilibrium that compartment is no more a part of the system. If, in addition, the particular equilibrium state is stable, then the corresponding subsystem goes to extinction. This also happens in the case of any one of the equilibrium conditions (6)–(10) being violated: zero M_i^* values are interpreted as before; negative M_i^* values signify that the subsystem i shows a tendency to separate from the system. The distintegration in question will take place when M_i approaches zero, as long as the corresponding steady state is stable.

The above illustrates the importance of stability analysis for the investigation of the system's dynamics. By linearization of eqns (4) in the

TABLE 1
Summary of Equilibrium and Stability Conditions for All Possible Equilibria of the Biomass System

	Equilibria[a]						*Constraints*[b]					
Sectors	1	2	3	4	5	0	1	2	3	4	5	0
I	M_1^*	M_2^*	M_3^*	M_4^*	M_5^*	M_0^*	(7)	(6)	(12)–(13)	—	(9)	(10)
II	0	M_2^*	M_3^*	M_4^*	M_5^*	M_0^*	$A_2 > \frac{q_2}{a_{12}} A_1$	$A_2 > 0$	(12)–(13)	—	(9)	(10)
III	M_1^*	0	M_3^*	M_4^*	M_5^*	M_0^*	$A_2 < -\frac{a_{12}}{q1} A_1$	(6)	(12)–(13)	—	(9)	(10)
IV	M_1^*	M_2^*	M_3^*	M_4^*	0	M_0^*	(7)	(6)	(12)–(13)	—	$A_5 < 0$	(10)
V	0	0	M_3^*	M_4^*	M_5^*	M_0^*	$A_1 < 0$	$A_2 < 0$	$A_3 > 0$, unstable	—	(9)	(10)
VI	0	M_2^*	M_3^*	M_4^*	0	M_0^*	$A_2 > \frac{q_2}{a_{12}} A_1$	$A_2 > 0$	(12)–(13)	—	$A_5 < 0$	(10)
VII	M_1^*	0	M_3^*	M_4^*	0	M_0^*	$A_2 < -\frac{a_{12}}{q_1} A_1$	(6)	(12)	—	$A_5 < 0$	(10)
VIII	0	0	any $M_3 > 0$	any $M_4 > 0$	0	any $M_0 > 0$	$A_1 < 0$	$A_2 < 0$	$A_3 = 0$, unstable	—	$A_5 < 0$	—
IX	0	0	0	0	0	0	$A_1 < 0$	$A_2 < 0$	(12)	—	$A_5 < 0$	—

[a] M_i^* values calculated by eqns (5a–f).
[b] Numbers in parentheses refer to corresponding relationships in the text.

vicinity of M_i^* according to the Liapunov indirect method,[28] the following characteristic equation of the system is obtained:

$$\begin{vmatrix} B_{11}-b & B_{12} & 0 & 0 & 0 & 0 \\ B_{21} & B_{22}-b & 0 & 0 & 0 & 0 \\ B_{31} & B_{32} & B_{33}-b & 0 & B_{35} & 0 \\ B_{41} & B_{42} & B_{43} & B_{44}-b & B_{45} & 0 \\ 0 & 0 & 0 & 0 & B_{55}-b & 0 \\ B_{61} & B_{62} & B_{63} & B_{64} & B_{65} & B_{66}-b \end{vmatrix} = 0 \quad (11a)$$

which is equivalent to

$$[b^2-(B_{11}+B_{22})\,b-B_{12}B_{21}](B_{33}-b)(B_{44}-b)(B_{55}-b)(B_{66}-b)=0 \quad (11b)$$

The quantities B_{ij} are functions of the coefficients a_{ij}, p_i and q_i, as defined in Appendix II.

By solving eqn (11b) for b, we obtain the six eigenvalues of the system. The equilibrium given by eqn (5) will be stable if all eigenvalues are real and negative. In this case, the system will tend to asymptotically approach equilibrium from one side with time. The stability conditions, which must be satisfied along with the equilibrium conditions (6)–(10), are easily obtained from (11b) and (27):

$$A_3<0 \quad (12)$$

which, because of eqn (8), is equivalent to

$$F_3^*(M)<0 \quad (13)$$

and

$$(M_1^*-\rho_1 M_2^*)(M_1^*-\rho_2 M_2^*)\geqq 0 \quad (14)$$

where

$$\rho_{1,2}=\left[\frac{a_{12}\pm\sqrt{(a_{12}^2+q_1q_2)}}{q_1}\right]^2>0 \quad (15)$$

Violation of the two stability constraints by the system's coefficients affects the dynamics of the system in two different ways (Fig. 2):

(a) When the left-hand side of eqn (13) becomes positive, e.g. due to the relatively high equilibrium value of human population, one eigenvalue becomes positive and the equilibrium M_i^* is unstable.

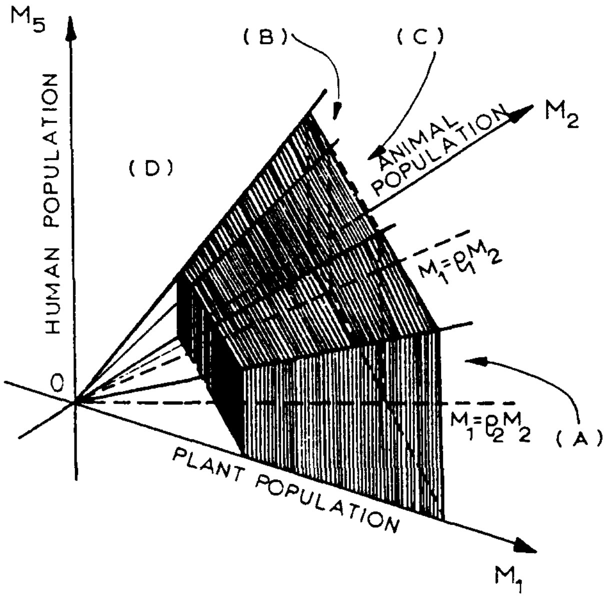

Fig. 2. Graphical representation of the space of plant, animal and human populations which satisfy stability conditions (13) and (14): (A) and (B) asymptotically stable, (C) oscillatory stable, (D) unstable system.

(b) When the left-hand side of eqn (14) becomes negative, i.e. when the ratio M_1^*/M_2^* of the equilibrium values of plant and animal populations is kept between ρ_1 and ρ_2, two eigenvalues are complex numbers with a negative real part; then diminishing oscillations will appear around M_i^*, so that the system will still tend to approach equilibrium with time.

Therefore, eqn (12) or (13) constitutes the only true stability condition for the non-zero equilibrium M_i^*. The stability constraints for the other possible equilibria of the system are summarized in Table 1.

DISCUSSION

The question of stability of equilibria including zero values of M_i is critical for the dynamics of the integrated system proposed in this work, since it affects the tendency for disintegration. Table 1 shows that there are six zero-containing equilibria, which can be stable provided that the corresponding constraints are satisfied:

in II there is no plant-biomass production.
in III there is no animal-biomass production,

in IV there is no human community supported by the system,
in VI the system is limited to animal farming and processing of animal products,
in VII the system is limited to plant-biomass production and processing, and finally
IX represents the total failure of the system.

From Table 1 it is obvious that the system cannot have two stable equilibria for the same values of its coefficients. Consequently, constraints (6), (7), (9), (10), (12) and (13) constitute the unique set of conditions for the existence of an integrated biomass system, under the assumptions of this work.

Of particular interest are the potential effects of biomass utilization taking place at level 4 on the overall dynamics. It is believed that the addition of one more consumer contributes to the stability of the system, due among other things to an expected increase in controllability and especially in connectivity, which decreases sensitivity to parameter variation.[25,28–30] For example, the presence of the term a_{34} in A_3 (25c) acts in favor of stability according to constraint (12).

By selecting the appropriate biomass conversion technologies to be applied at level 4, we can add the resulting flows of energy, chemicals and novel organics to the network shown in Fig. 1:

$$a_{4s}M_4 = \sum_i a_{4i}M_i \text{ (recipient control)}, \ a_{4i} > 0 \tag{16}$$

$$A_i' = A_i + a_{4i} > A_i \quad (i = 1, 2, 3 \text{ and } 5) \tag{17}$$

If eqn (17) is applied to constraints (6) and (7), it becomes evident that biomass utilization can significantly promote the integration of plant, animal and human communities.

When modification (16) is introduced to the model, an additional equilibrium condition is necessary:

$$(a_{14} - a_{41})\, M_1^* + (a_{24} - a_{42})\, M_2^* + (a_{34} - a_{43})\, M_3^* + (a_{54} - a_{45})\, M_5^* > 0 \tag{18}$$

This relationship can also be viewed as a biomass availability constraint to be satisfied by the coefficients of the system. Violation of eqn (18) means that, when the system reaches a steady state, there will be no available biomass quantity for conversion.

According to eqn (16), subsystem 4 also behaves as a producer of energy and various organic products for the system. Theoretical ecology teaches that the introduction of one more producer could have negative results on the overall dynamics.[27] Such effects can be predicted by the proposed model, e.g. when excessive quantities of biomass are absorbed through the terms $\dot{m}_{14}$ and $\dot{m}_{24}$ thus limiting the amount available for recycling to the soil and other uses. However, even in those extreme situations, the system can always reach an equilibrium (accordingly displaced) which will still be stable as long as

$$A_3' = a_{x3} - a_{34} - a_{30} + a_{43} < 0 \quad \text{or} \quad a_{34} - a_{43} > a_{x3} - a_{30} \qquad (19)$$

Obviously, the system has been supplied by formation with substantial, built-in, self-stabilizing capacity. This is mainly the result of the introduction of density effect terms, $q_i M_i^2$ ($i = 1, 2, 5$), as explained above. Stability requires that q_5 and one of q_1, q_2 have to be positive, otherwise the non-zero equilibrium becomes unstable and the integration of plant, animal and human populations is impossible.

An additional observation refers to plant–animal interaction, as expressed by a_{12}. The preceding analysis shows that this coefficient plays a significant role in determining the dynamic behavior of the biomass system. In the special case when $a_{12} = 0$, i.e. the two compartments are disconnected, the possibility for oscillation around steady state is eliminated; the system becomes asymptotically stable as long as condition (13) is satisfied. If, on the other hand, the particular interaction is controlled by one of the levels (donor or recipient control), then two non-zero equilibria appear, only one of them stable.

APPLICATION OF THE MODEL — AN EXAMPLE

In general, any application of the modeling approach proposed by this paper for the analysis of complicated productive biosystems should follow the following three steps:

- Step 1. The system under investigation is clearly defined and described as in Fig. 1. Then its mathematical representation is derived in the form of differential rate equations, as was done with eqns (4).
- Step 2. The values of the model's coefficients a_{ij}, p_i, q_i or their lumped expressions A_i are determined by statistical methods, i.e. fitting data in the form $M_i = M_i(t)$ by the equations of the model over a certain period of time, for which the coefficients in question can be assumed to keep constant values.
- Step 3. The dynamics of the system are quantitatively analyzed with the help of the differential equations of the model; emphasis should be given to the existence of zero and non-zero equilibrium states

and their stability, as well as to the effects of biomass utilization thereon.

In an effort to illustrate the potential of this approach and, at the same time, indirectly validate the major assumptions of this work, we have considered the following example:

System

Greek agriculture and agro-industry for the period 1960–73, characterized by a rapid growth without any significant structural changes;[31] therefore, relatively constant values can be anticipated for coefficients A_i.

Model

Five typical components of the system were included in the investigation, i.e. grain maize (M_1), poultry (M_2), total biomass available as field residues (M_4), human population (M_5), and soil fertility, as measured by the nutrients and organics content of the top soil (M_0). The equations used were derived from eqn (4) for $a_{12} = 0$ and recycling only in the form of limited intrinsic (biological) return of phytomass to the soil.

Fitting

Yearly data on M_1, M_2 and M_5 and estimates of M_4 and M_0 were taken from the literature.[23,31–33] Fitting was based on the least-squares method.[34] The results are summarized in Fig. 3. The analytic expressions of the equations are presented in Appendix III. The correlation coefficients determined are in the area 90–98%, with an average of 95%, which is considered as satisfactory.

Analysis

The system appears to have departed from an original equilibrium state around 1960; that situation, when judged according to the criteria of the following period, is found unstable. The productive potential of the system, after a period of rapid growth, tends to level off so that the system, appears to have asymptotically approached a second equilibrium state in the mid 1970s. This new equilibrium is stable for all subsystems with the exception of soil fertility, M_0. In the latter case, constraint (10) above seems to be constantly violated; this results in an explosive loss of

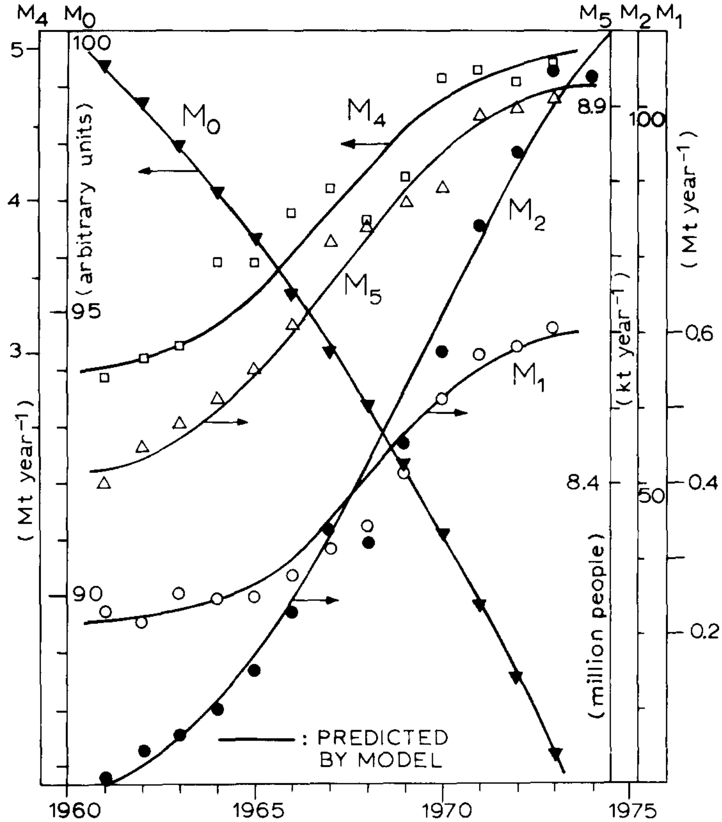

Fig. 3. Application of the model to Greek agriculture (1960–73). M_1 is grain maize production in Mt year^{-1} (○),[32] M_2 is poultry meat production in kt year^{-1} (●),[32] M_4 is plant biomass available as field residues in Mt year^{-1} (□),[31] M_5 is human population in millions (△),[33] M_0 is soil fertility in arbitrary % units (▼).[23]

potential productive capacity which in the long run threatens the existence of the system, while in the short run it implicates an increasing use of fertilizers.

It should be added at this point that, due to the oil crisis and the major economic, socio-political and environmental developments of the 1970s, the system examined here has entered a new phase following the one analyzed above; this phase is presently under investigation by our group.

Biomass utilization in this system could have a prospect provided that a significant part of the plant-biomass available in the form of field and agro-industrial residues is technically recycled to restore the productive potential of the system. This objective can be achieved in many ways, according to various technological paths, e.g. ploughing under, erosion protection, composting, biogas production and recycling of the residue, feeding animals and recycling of the manure etc. In all cases, stability can be obtained only at the expense of the system's surplus biomass, thus limiting the potential availability of bioresources for conversion to biofuels, biochemicals and biomaterials.

APPENDIX I: MASS-ENERGY RELATIONSHIPS

The various energy quantities appearing in the energy balance equations (2) can be related to the corresponding mass quantities as follows:

$$E_j = \lambda_j M_j \tag{20a}$$

$$\dot{e}_{ji} = \lambda_{ji} \dot{m}_{ji} \tag{20b}$$

$$\lambda_{01} = \varepsilon_0 \tag{20c}$$

$$\dot{e}_{i0} = \varepsilon_i \dot{m}_{i0} \tag{20d}$$

$$\dot{e}_{i0}^{D} = \lambda_i \dot{m}_{i0}^{D} \tag{20e}$$

$$\dot{e}_{xj} = \varepsilon_j' \dot{m}_{xj} \tag{20f}$$

where i and j are as in eqn (1); λ_i is the energy content of biomass M_j; λ_{ji} is the energy content of the mass flow $\dot{m}_{ij}$; ε_j and ε_j' are the process energy requirements or losses per unit of mass input or output, depending on the case. We can furthermore assume that

$$\lambda_{ji} = \lambda_j \quad (j \neq 0) \tag{21}$$

By using relationships (20) and (21) to transform the energy equations (2), and then by subtracting the corresponding mass equations at levels 1 and 5, we obtain

$$\dot{m}_{01} = \frac{\lambda_1 - \varepsilon_1}{\lambda_1 - \varepsilon_0} \dot{m}_{10} \tag{22a}$$

$$\dot{m}_{35} = \frac{\lambda_5 - \varepsilon_5}{\lambda_5 - \lambda_3} \dot{m}_{50} \tag{22b}$$

Since the technical recycle flows can be assumed to be proportional to the biomass of the donor level

$$\dot{m}_{i0} = a_{i0} M_i \tag{23}$$

then the expressions (22) take the following form:

$$\dot{m}_{01} = a_{01}M_1, \quad a_{01} = \frac{\lambda_1 - \varepsilon_1}{\lambda_1 - \varepsilon_0}\, a_{10} \tag{24a}$$

$$\dot{m}_{35} = a_{35}M_5, \quad a_{35} = \frac{\lambda_5 - \varepsilon_5}{\lambda_5 - \lambda_3}\, a_{50} \tag{24b}$$

It must be noted that the linear form of eqns (24a) and (24b) is the result of the combination of mass and energy equations under the assumptions (20), (21) and (23); no particular assumption about the ecological interrelation[27] between levels 0 and 1, and 3 and 5, respectively, is necessary. The expressions obtained indicate the existence of a certain effect of dominance of the one level on the other, which can be generalized (see text, eqn (3)).

APPENDIX II: DEFINITIONS

The simplify the mathematical formulation of the model, we have defined the following quantities and functions:

$$A_1 = a_{01} - a_{13} - a_{14} - a_{10} - p_1 \tag{25a}$$

$$A_2 = a_{x2} + a_{32} - a_{23} - a_{24} - a_{20} - p_2 \tag{25b}$$

$$A_3 = a_{x3} - a_{34} - a_{30} \tag{25c}$$

$$A_4 = a_{4s} + a_{40} \tag{25d}$$

$$A_5 = a_{35} - a_{54} - a_{50} - p_5 \tag{25e}$$

$$A_0 = a_{x0} - a_{01} \tag{25f}$$

$$F_3(M) = a_{35}M_5 + (a_{32} - a_{23})\, M_2 - a_{13}M_1 \tag{26a}$$

$$F_4(M) = \sum_i a_{i4}M_i \quad (i = 1, 2, 3, 5) \tag{26b}$$

$$F_0(M) = \sum_j a_{j0}M_j + \sum_l (p_l M_l + q_l M_l^2) \quad (j = 1\text{–}5;\ l = 1, 2, 5) \tag{26c}$$

$$B_{11} = A_1 - 2q_1M_1^* - a_{12}M_2^* \tag{27a}$$

$$B_{22} = A_2 + a_{12}M_1^* - 2q_2M_2^* \tag{27b}$$

$$B_{12} = -\, a_{12}M_1^* \tag{27c}$$

$$B_{21} = a_{12}M_2^* \tag{27d}$$

$$B_{33} = A_3 \tag{27e}$$

$$B_{44} = -A_4 \tag{27f}$$

$$B_{55} = A_5 - 2q_5 M_5^* \tag{27g}$$

$$B_{66} = -a_{0x} \tag{27h}$$

APPENDIX III: APPLICATION TO GREEK AGRICULTURE (1960–73)

The general form of the differential equations of the model in this case is the following:

$$\frac{dM_i}{dt} = aM_i\ bM_1^2 \tag{28}$$

Integration of eqn (28) gives

$$M_i = \frac{-a/b}{1 - ((a/b)/C_1)\,e^{-at}} \tag{29}$$

where C_1 is an integration constant and t the time in years.

Fitting production and other data shown in Fig. 3 into eqn (29) and taking 1960 as the base year in all cases ($t = 0$), we obtain the following equations:

$$M_1 = 210 + \frac{406}{1 + 113{\cdot}3\ \exp(-0{\cdot}584t)}\ (\text{kt year}^{-1}\text{, grain maize}) \tag{30a}$$

$$M_2 = \frac{134{\cdot}8}{1 + 18{\cdot}06\ \exp(-0{\cdot}303t)}\ (\text{kt year}^{-1}\text{, poultry meat}) \tag{30b}$$

$$M_4 = 2{\cdot}80 + \frac{2{\cdot}22}{1 + 43{\cdot}25\ \exp(-0{\cdot}524t)}\ (\text{Mt year}^{-1}\text{, residue biomass}) \tag{30c}$$

$$M_5 = 8{\cdot}39 + \frac{0{\cdot}560}{1 + 31{\cdot}01\ \exp(-0{\cdot}481t)}\ (\text{M, human population}) \tag{30d}$$

$$M_0 = 0{\cdot}6 + \frac{0{\cdot}8}{1 + 20{\cdot}67\ \exp(-0{\cdot}416t)}\ (\text{\% per year, fertility loss}) \tag{30e}$$

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